X. *A Method of extracting the Root of an Infinite Equation.* By A. De Moivre, *F. R. S.*

Theorem.

If $az + bzz + cz^3 + dz^4 + ez^5 + fz^6$, &c. $= gy + hyy + iy^3 + ky^4 + ly^5 + my^6$, &c. then will z be $= \frac{g}{a}y + \frac{h-bAA}{a}y^2$

$$+ \frac{i-2bAB-cA^3}{a}y^3$$

$$+ \frac{k-bBB-2bAC-3cAAB-dA^4}{a}y^4$$

$$+ \frac{l-2bBC-2bAD-3cABB-3cAAC-4dA^3B-eA^5}{a}y^5$$

$$+ \frac{m-2bBD-bCC-2bAE-cB^3-6cABC-3cAAD-6dAABB-4dA^3C-5eA^4B-fA^6}{a}y^6 \text{ \&c.}$$

For the underſtanding of this Series, and in order to continue it as far as we pleaſe; it is to be obſerved, 1. That every Capital Letter is equal to the Coefficient of each preceding Term; thus the Letter B is equal to the Coefficient $\frac{h-bAA}{a}$. 2. That the Denominator of each Coefficient is always a. 3. That the firſt Member of each Numerator, is always a Coefficient of the Series $gy + hyy + iy^3$, &c. *viz.* the Firſt Numerator begins with the firſt Coefficient g, the Second Numerator with the Second Co-efficient h, and ſo on. 4. That in every Member after the Firſt, the Sum of the Exponents of the Capital Letters, is always equal to the Index of the Power to which this Member belongs: Thus conſidering the Coefficient $\frac{k-bBB-2bAC-3cAAB-dA^4}{a}$, which belongs to the Power y^4, we ſhall ſee that in every Member bBB, $2bAC$ $3CAAB$, dA^4, the Sum of the Exponents of the Capital Letters is 4, (where I muſt take notice, that by the Exponent of

a

a Letter; I mean the Number which expresses what Place it has in the Alphabet; thus 4 is the Exponent of the Letter *D*) hence I derive this Rule for finding the Capital Letters of all the Members that belong to any Power; *Combine the Capital Letters as often as you can make the Sum of their Exponents Equal to the Index of the Power to which they belong.* 5. That the Exponents of the small Letters, which are written before the Capitals, express how many Capitals there is in each Member. 6. That the Numerical Figures or *Unciæ* that occur in these Members, express the Number of Permutations which the Capital Letters of every Member are capable of.

For the Demonstration of this; suppose $z = Ay + Byy + Cy^3 + Dy^4$, *&c.* Substitute this Series in the room of z, and the Powers of this Series, in the room of the Powers of z; there will arise a new Series; then take the Co-efficients which belong to the several Powers of y, in this new Series, and make them equal to the corresponding Co-efficients of the Series $gy + hyy + iy^3$, *&c.* and the Co-efficients A, B, C, D, *&c.* will be found such as I have determined them.

But if any one desires to be satisfied, that the Law by which the Co-efficients are form'd, will always hold, I'll desire 'em to have recourse to the Theorem I have given for raising an infinite Series to any Power, or extracting any Root of the same; for if they make use of it, for taking successively the Powers of $Ay + Byy + Cy^3$, *&c.* they will see that it must of necessity be so. I might have made the Theorem I give here, much more general than it is; for I might have suppos'd,

$az^m + bz^{m+1} + cz^{m+2}$ *&c.* $= gy^m + hy^{m+1} + iy^{m+2}$ *&c.* then all the Powers of the Series $Ay + Byy + Cy^3$, *&c.* design'd by the universal Indices, must have been taken successively; but those who will please to try this, my easily do it, by means of the *Theorem for raising an infinite Series to any Power*, &c.

This *Theorem* may be applied to what is called the Reversion of Series, such as finding the Number from its Logarithm given; the Sine from the Arc; the Ordinate of an Ellipse from an Area given to be cut from any Point in the Axis: But to make a particular Application of it, I'll suppose we have this Problem to solve; *viz.* The Chord of an Are being

ing given, to find the Chord of another Arc, that ſhall be to the firſt as n to 1. Let y be the Chord given, z the Chord required; now the Arc belonging to the Chord y is, $y + \frac{y^3}{6dd} + \frac{3y^5}{40d^4} + \frac{5y^7}{112d^6}$ &c. and the Arc belonging to the Chord z is $z + \frac{z^3}{6dd} + \frac{3z^5}{40d^4} + \frac{5z^7}{112d^6}$ &c. the firſt of theſe Arcs is to the ſecond as 1 to n; therefore multiplying the Extreams and Means together, we ſhall have this Equation:

$$z + \frac{z^3}{6dd} + \frac{3z^5}{40d^4} + \frac{5z^7}{112d^6} \text{ \&c.} = ny + \frac{ny^3}{6dd} + \frac{3ny^5}{40d^4} + \frac{5ny^7}{112d^6} \text{ \&c.}$$

Compare theſe Two Series with the Two Series of the *Theorem*, and you will find $a = 1$, $b = 0$, $c = \frac{1}{6dd}$, $d = 0$, $e = \frac{3}{40d^4}$ $f = 0$, &c. $g = n$, $h = 0$, $i = \frac{n}{6dd}$, $k = 0$, $l = \frac{3n}{40d^4}$, $m = 0$, &c. hence z will be $= ny + \frac{n - n^3}{6dd} y^3$ &c. or $ny + \frac{1 - nn}{2 \times 3dd} yyA$, &c. Suppoſing A to denote the whole preceding Term, which will be the ſame Series as Mr. *Newton* has firſt found.

By the ſame Method, this general Problem may be ſolv'd; the Abſciſſe correſponding to a certain Area in any Curve being given, to find the Abſciſſe, whoſe correſponding Area ſhall be to the firſt in a given Ratio.

The Logarithmic Series might alſo be found without borrowing any other Idea, than that Logarithms are the Indices of Powers: Let the Number, whoſe Logarithm we inquire, be $1 + z$, ſuppoſe its Log. to be $az + bzz + cz^3$, &c. Let there be another Number $1 + y$; thereof its Logarithm will be $ay + byy + cy^3$, &c. Now if $1 + z = \overline{1 + y}|^n$, it follows, that $az + bzz + cz^3$ &c, $ay + byy + cy^3$, &c :: n, 1. that is, $az + bzz + cz^3$, &c. $= nay + nbyy + ncy^3$, &c,

Therfore

Therefore we may find a Value of z exprest by the Powers of y; again, since $1+z=\overline{1+y}|^{n}$, therefore $z=\overline{1+y}|^{n}-1$ that is $z=ny+\frac{n}{1}\times\frac{n-1}{2}yy+\frac{n}{1}\times\frac{n-1}{2}\times\frac{n-2}{3}y^3$, *&c.* Therefore z is doubly exprest by the Powers of y. Compare these Two Values together, and the Co-efficients a, b, c, *&c.* will be determin'd, except the first a which may be taken at Pleasure, and gives accordingly, all the different Species of Logarithms.

XI. *An Account of the Appearance of an extraordinary* Iris *seen at* Chester, *in* August *last, by* E. Halley.

ON the Sixth Day of *August* last, in the Evening, between Six and Seven of the Clock, I went to take the Air upon the Walls of *Chester*, when I was surprized by a sudden Shower, which forced me to take Shelter in a Nich that afforded me a Seat in the Wall, near the North East Corner thereof. As I sat there, I observed an *Iris*, exceedingly vivid, as to its Colours, at first on the South Side only, but in a little Time with an entire Arch; and soon after, the Beams of the Sun being very strong, there appeared a secondary *Iris*, whose Colours were more than ordinary Bright; but inverted, as usually: that is, the Red was inwards, which in the primary *Iris* is outward, and *è contra* for the Blues. But what I took most Notice of was, that with these Two concentrick Arches, there appeared a Third Arch, near upon as bright as the Secondary *Iris*, but coloured in the Order of the Primary, which took its Rise from the Intersection of the Horizon and Primary *Iris*, and went